Rich activities

<https://www.ncetm.org.uk/enquiry/16956>

**Starting Point - Exploring rich mathematical tasks**

**An investigation into what constitutes a rich mathematical task and what kind of tools, resources and classroom practice can support the use of such tasks in classrooms**

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**Research and Discussion**

The concept of a rich mathematical task is a fairly new one, often used without definition. There is relatively little research on rich tasks as such. However, a good deal of research strongly suggests that mathematical tasks that we refer to as "rich", are those that are most likely to engage learners positively and effectively with their mathematical learning.

**What do we mean by a rich task?**

The concept of a “rich task” probably goes back to Afzal Ahmed. In 1987 he offered a set of criteria for identifying a "Rich Mathematical Activity" :

* must be accessible to everyone at the start
* needs to allow further challenges and be extendable
* should invite learners to make decisions
* should involve learners in speculating, hypothesis making and testing, proving or explaining, reflecting and interpreting
* should not restrict learners from searching in other directions
* should promote discussion and communication
* should encourage originality and invention
* should encourage "what if?" and "what if not?" questions
* should have an element of surprise
* should be enjoyable.

(adapted from page 20 of "Better Mathematics: A Curriculum Development Study"[a])

Although these criteria are not based on research, they have had an important influence on the subsequent development of rich tasks, for example in Malcolm Swan’s work "Improving Learning in Mathematics: challenges and strategies"[b]. Here he contrasts simple and rich tasks, emphasising the importance of challenge for developing mathematical reasoning noting that :

*“Textbooks often assume that we should begin topics by solving simple questions and then gradually move towards more complex questions. While this may appear natural, we find that learners tend to solve simple questions by intuitive methods that do not generalise to more complex problems. When the teacher insists that they use more generalisable methods, learners do not understand why they should do so when intuitive methods work so well. Simple tasks do not motivate a need to learn.*

*Rich tasks also allow all learners to find something challenging and at an appropriate level to work on.”*

**What research is available on rich tasks?**

In recent years there has been a renewed interest in designing tasks that foster collaboration and discussion about mathematics amongst learners. As with most areas of developing research there are a number of themes that are being explored, a few of which are identified here.

**Typology of rich tasks**
There is an aspect of Malcolm Swan’s work that is an exploration of task design because as well as well as identifying eight principles of effective mathematics teaching, he has also attempted to produce a typology of activities that support these principles. In “Collaborative Learning in Mathematics: A challenge to our beliefs and practices” (pages 145-150) he classifies the activities as five types:

* Comparing representations
* Evaluating the validity of statements and generalisations
* Correcting and diagnosing common mistakes
* Resolving problems that generate cognitive conflict
* Creating problems and connections between concepts and representations.

In "Improving Learning in Mathematics: challenges and strategies"[b] (page 16) he offers a different typology. Practicing teachers could act as researchers in this area, try the different activities and write a report on which is the more appropriate typology. It would also be interesting to discuss whether or not the typology has value.

**Mathematics in a micro-world**
The constructionist paradigm of Papert leads to the idea of a microworld. This provides an environment for solving problems in which learners can experience the constraints of the underlying mathematical system and so construct their own system. For example, a piece of mathematical knowledge may be incorporated into an item of software, which forms a part, or the whole of a micro-world environment. By acting and interacting with the software a mathematical concept emerges from the set of problems to which it provides a means of solution.

The use of software to create microworld environments is a relatively new, but growing, area of interest for the location of rich tasks. Some of the issues are outlined in “The Influence of technological advances on mathematics learning” by M A Marriotti. This is in a section which looks at the influences of advanced technologies on mathematics education in the “Handbook of International Research in Mathematical Education” edited by Lyn D English (2002) which is worth reviewing.

**Realistic Mathematics Education (RME) and context**
The advocates of Realistic Mathematics Education suggest that the roots of mathematical understanding lie in mathematising real-world contexts and engaging in a process of guided re-invention. The RME project started with the work of members of the Freudenthal Institute who developed principles for the design of RME curricular, or learning trajectories. The characteristics of RME applied to a lesson involve taking an intended topic and situating it in reality in order to give it context. The context serves as a source and an area of application. The task may intertwine more than one area of mathematics e.g. fractions and proportions because during the learning process the aim is to produce tools through collective effort. The tools take the form of symbols, diagrams and situation or context models that give the mathematics meaning.

The process is outlined in “[How to Design Mathematics Lessons based on the Realistic Approach](http://www.geocities.com/ratuilma/rme.html%22%20%5Cl%20%22anchor73250%22%20%5Ct%20%22_blank)”. There is also a growing set of reports by teachers of their attempts to use this approach e.g. “[Teaching and learning in realistic mathematics education](http://www.partnership.mmu.ac.uk/cme/Student_Writings/TS1/MarkHerman.html%22%20%5Ct%20%22_blank)” by Mark Hermann.

**Planning the use of rich tasks**
Some difficulties associated with using RME are raised by Ainley, J., Pratt, D. and Hansen, A[c] who nevertheless identify the value of the approach in two characteristics that they describe as “purpose” and “utility”.

By purpose they mean that a task has a meaningful outcome for the learner, “in terms of an actual or virtual product, or the solution of an engaging problem.” This is not the same as contextualising mathematics in supposedly real-world settings, because there is evidence that these attempts “fail to provide purpose to which the learner can relate, either in terms of the overall task, or the ways in which mathematical ideas are used within it.”

The second characteristic is utility. By this they mean “that the learning of mathematics encompasses not just the ability to carry out procedures, but the construction of meaning for the ways in which those mathematical ideas are useful.”

By planning the use of tasks which embody purpose and utility we can avoid two common problems with task design. The first is the use of tasks that are tightly focused on curriculum objectives, which often result in tasks that are unrewarding for learners. The second problem arises when we concentrate on finding engaging activities but their completion often has an unfocussed feel to them and are difficult to assess. As the authors say, “Designing tasks that are purposeful for learners ensures that the activity will be rich and motivating. Such purposeful tasks provide opportunities to learn about the utility of particular ideas, which will give the focus that may otherwise be absent.”

**Where can I find examples of rich tasks?**

**Improving Learning in Mathematics**The approaches involved in the Standards Unit post-16 project ‘Improving Learning in Mathematics’ are very much based on the concept of using rich, collaborative mathematical tasks. The development of these followed on from the research carried out by Malcolm Swan, who focused on the transformation of mathematics classrooms through the design of research-based approaches and materials. He considered research-based principles for effective teaching and learning in mathematics, and "put these to work" in the classroom. In his book, "Collaborative Learning in Mathematics: a challenge to our beliefs and practices", he describes the impact of these approaches on the professional development of more than forty teachers. He evaluates their changed beliefs and practices and the resulting effects on learning.

An Ofsted report, "Evaluating provision for 14-19 year-olds" (May 2006, Ref HMI 2611) endorsed his work with the then Department for Education and Skills (DfES) Standards Unit. The report concluded that one of the most significant contributing factors to high achievement in mathematics is:

* Teaching that focuses on developing students’ understanding of mathematical concepts and enhances their critical thinking and reasoning, together with a spirit of collaborative enquiry that promotes mathematical discussion and debate.

The development of such teaching is at the core of Swan’s work and is exemplified in the resources developed for *Improving Learning in Mathematics*.

**NRICH**
The mathematical enrichment web site [NRICH](http://nrich.maths.org) has been going for several years now and has a large quantity of enrichment activities. NRICH is a team of teachers of mathematics that works with the Faculties of Education and of Mathematics at the University of Cambridge. The team has been involved in the most recent development of the National Curriculum.

NRICH has five aims:

* Enrich the experience of the mathematics curriculum for all learners
* Offer challenging and engaging activities
* Develop mathematical thinking and problem-solving skills
* Show rich mathematics in meaningful contexts
* Work in partnership with teachers, schools and other educational settings

Originally, NRICH was designed to provide stimulating materials for the mathematically gifted throughout the 5-19 age range. However, the site has since widened its remit and provides for the full range of abilities. It is not focussed exclusively on rich mathematical tasks but, as the five aims above indicate, enriching the mathematical experience of learners.

NRICH publishes its [own research](http://nrich.maths.org/public/viewer.php?obj_id=2719&part=) on the effectiveness of the methods used and promoted by the NRICH site and most of the papers are available online. There is an introductory article by [Jennifer Piggott on Rich Tasks](http://nrich.maths.org/public/viewer.php?obj_id=5662&part=) as well as a set of [differentiated CPD resources on Integrating Rich Tasks](http://nrich.maths.org/public/viewer.php?obj_id=6089&part=).

There are other sites that seek to provide a range of rich mathematical activities of varying usefulness. Three sites that have a good range of rich tasks are:

* The Freudenthal Institute's [WisWeb](http://www.fi.uu.nl/wisweb/en/) has a large range of rich online activities for learners of all ages and abilities
* Jonny Griffiths' [Rich Starting Points (RISPS)](http://www.risps.co.uk/) for A Level Core Mathematics (originally funded by a Gatsby Teacher Fellowship)
* a related site set up by Jonny Griffiths [Making Statistics Vital](http://www.making-statistics-vital.co.uk/)

**NCETM Portal Resources**

This is a selection of resources available on the [NCETM portal](https://www.ncetm.org.uk/) that relate to what constitutes a rich mathematical task and what kinds of tools, resources and classroom practice can support the use of such tasks in classrooms

**Teacher Enquiry**

* [Improving the achievement and motivation of KS3 pupils in mathematics lessons – Failsworth School](https://www.ncetm.org.uk/enquiry/6328)
* [The Octagon Project – Harlow CDC](https://www.ncetm.org.uk/enquiry/5486)
* [Using NCETM Self-evaluation Tools to support professional development in Using and Applying mathematics within KS2 and KS3 – University of Cumbria](https://www.ncetm.org.uk/enquiry/12149)

**Resources**

* [East Midlands Creating Mathematics Networks - Amber Valley Network](https://www.ncetm.org.uk/resources/10428)
* [Online presentation about ICT and rich tasks](https://www.ncetm.org.uk/public/files/260007/Workshop%2BC5.ppt)
* [Online presentation about mediating web-based rich tasks](https://www.ncetm.org.uk/public/files/259341/Workshop%2BA3%2B-%2BMediating%2Bweb%2Bbased.ppt)
* [Rich Maths Tasks to Engage at KS3](https://www.ncetm.org.uk/resources/15409) – link to the [East Midlands Maths site](http://emmaths.jfcs.org.uk/)
* There are a large number of resource links to the [nrich](http://nrich.maths.org/) website. This page is a typical example:
* [NCETM Secondary Magazine, Issue 2 - Website of the week](https://www.ncetm.org.uk/resources/8025)

**Courses & Training**

* [Mathematics Enrichment, problem solving and rich tasks a suite of nrich on-request courses](https://www.ncetm.org.uk/cpd/2162)
* [Rich Tasks](https://www.ncetm.org.uk/cpd/6176) a flexible course delivered by nrich for Key Stages 2, 3 and 4

**Mathemapedia**

* [Rich Tasks](https://www.ncetm.org.uk/mathemapedia/Rich%20Tasks)
* [Using rich collaborative tasks - AS and A2](https://www.ncetm.org.uk/mathemapedia/Using%20rich%20collaboative%20tasks%20-%20AS%20and%20A2)

**Community**

**Communities**There are no communities that deal specifically with rich tasks. Here are a couple of links to discussions about rich tasks in the Maths Café:

* [functional skills (and rich tasks)](https://www.ncetm.org.uk/community/thread/36220#com36226)
* [What is understanding? – Revisited](https://www.ncetm.org.uk/community/thread/18814#com20058)

**Blogs**

* [Reflections - Rich tasks by Jennifer Piggott](https://www.ncetm.org.uk/blogs/2221)
* [Rich tasks - where in the SoW do we put them?](https://www.ncetm.org.uk/blogs/5826)
* [Ten million times mathematics](https://www.ncetm.org.uk/blogs/6105)

**References**

[a] Ahmed, A. (1987) *Better Mathematics: a curriculum development study* London: HMSO

[b] Swan, M (2005) *Improving Learning in Mathematics: challenges and strategies* DfES Standards Unit Publication

[b] Ernest, P. (2002) "Empowerment in Mathematics Education" *Philosophy of Mathematics Education Journal* 15 (available online)

[c] Ainley, J., Pratt, D. and Hansen, A. (2006), Connecting engagement and focus in pedagogic task design, British Educational Research Journal. 32.1, 23-38