Activité NRICH

1er cycle du secondaire

**Des fractions au milieu**

Je cherche des fractions dont les valeurs se situent entre et . On me dit qu’il y a une infinité de bonnes réponses.

Trouve autant de réponses que possible.

Trouve des réponses dont le dénominateur est petit, tels que 10 ou 20.

Avez-vous trouvé des dénominateurs qui ne permettent aucune valeur entre et ? Démontre.

Source : <http://nrich.maths.org/344>

8e année

Le nombre

**Résultat d’apprentissage général :** Développer le sens du nombre.

**Résultat d’apprentissage spécifique**

RAS 2. Déterminer la racine carrée approximative d’un nombre qui n’est pas un carré parfait (se limitant aux nombres entiers positifs).

[C, CE, L, R, T]

[TIC : P2-3.4]

Mathématiques M-9 – Programme d’études de l’Alberta

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**Des fractions au milieu**

Solutions

We had lots of interesting responses to this problem. Ellie and Georgia from Melbourn Village College had a systematic way of finding fractions:  
  
http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58=7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png6158  to four decimal places.  
http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56=7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png4833  to four decimal places.  
We then converted the decimals betweenhttp://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56  and http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58  into fractions.  
For example, 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png55= . We simplified it by dividing the denominator and the numerator by 5, because 5 goes into both numbers. So one correct anwser is .  
  
Ellie and Georgia went on to find other fractions by converting decimals between 7.4833 and 7.6158.  
  
Ashley from Durrington High School and Ha Young Jung from Wesley College used a similar method to Ellie and Georgia to find lots of fractions between http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56 and http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58 .  
  
John from Takapuna Grammar School showed that all denominators apart from 1 and 3 will work (although he didn't explain how he knew 1 and 3 wouldn't work):  
  
First we calculate http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56  and http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58 , which turn out to be about 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png483 and 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png615 respectively. We can also eliminate quite a lot of denominators considering that any number x in which http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.pnghttp://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58−http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56  (which is about 0http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png132) has to have a fraction between the two numbers.  
  
Therefore we only need to test 1http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3B.png2http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3B.png3http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3B.png4http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3B.png5http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3B.png6 and 7.  
  
2http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3B.png4 and 6 can be easily explained off since (or 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png5) is between 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png483 and 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png615, and is the same as and . 5 is also explained off because (or 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png6) is also between 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png483 and 7http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3A.png615. lies in the range.  
  
  
Eduardo from the British School in Manila explained clearly why a denominator of 1 wouldn't work:  
http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.png http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.pnghttp://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58  so 56q2http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.pngp2http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.png58q2  
We know the denominator 1 will not work because there is no square number between 56 and 58, so when q=1, there is no value of p to satisfy the inequality.

Harry from the Beacon School used a similar algebraic approach to give a very clear and full solution.  
Let the fraction be .

Now http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.pnghttp://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.pnghttp://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58   
56http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.png http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.png58  
56y2http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.pngx2http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.png58y2  
  
Therefore, as x and y are whole numbers, there must be a square between 56y2 and 58y2 for a fraction between http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56  and http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58  to exist for denominator y.  
  
If there is no square in this interval, then the difference between the largest square below 56y2 and the smallest square above 58y2 must be greater than 2y2.  
  
The square root of the former must be the largest integer below http://nrich.maths.org/javascript/jsMath/fonts/cmex10/alpha/144/char70.png56y2   
  
The difference between n2 and the square immediately above it is:  
(n+1) 2−n2=2n+1  
  
So there is a solution in the interval when 2n+1http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.png2y2, where n is the largest integer below http://nrich.maths.org/javascript/jsMath/fonts/cmex10/alpha/144/char70.png56y2 .  
  
2http://nrich.maths.org/javascript/jsMath/fonts/cmex10/alpha/144/char70.png56y2+1http://nrich.maths.org/javascript/jsMath/fonts/cmmi10/alpha/144/char3C.png2y2  for y=8 but not y=7, so for yhttp://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char15.png8 there will always be a square between 56y2 and 58y2.  
  
For y=1, 56y2=56 and 58y2=58. There are no squares in this range, so 1 cannot be the denominator.  
  
For y=2, 56y2=224 and 58y2=232. There is a square in this range (225=152), so 2 can be the denominator. The numerator x is http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png225  so the fraction is   
  
For y=3, 56y2=504 and 58y2=522. There are no squares in this range, so 3 cannot be the denominator.  
  
For y=4, 56y2=896 and 58 y2=928. There is a square in this range (900=302), so 4 can be the denominator. The fraction is , which is equal to .  
  
For y=5, 56y2=1400 and 58y2=1450. There is a square in this range (1444=382), so 5 can be the denominator. The fraction is .  
  
For y=6, 56y2=2016 and 58y2=2088. There is a square in this range (2025=452), so 6 can be the denominator. The fraction is , which is equal to .  
  
For y=7, 56y2=2744 and 58y2=2842. There is a square in this range (2809=532), so 7 can be the denominator. The fraction is .  
  
There is a fraction between http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png56  and http://nrich.maths.org/javascript/jsMath/fonts/cmsy10/alpha/144/char70.png58  for all whole denominators except 1 and 3.  
  
Well done Harry!