Activité NRICH

1er cycle du secondaire

**Des fractions au milieu**

Je cherche des fractions dont les valeurs se situent entre $\sqrt{56}$ et $\sqrt{58}$. On me dit qu’il y a une infinité de bonnes réponses.

Trouve autant de réponses que possible.

Trouve des réponses dont le dénominateur est petit, tels que 10 ou 20.

Avez-vous trouvé des dénominateurs qui ne permettent aucune valeur entre $\sqrt{56}$ et $\sqrt{58}$ ? Démontre.

Source : <http://nrich.maths.org/344>

8e année

Le nombre

**Résultat d’apprentissage général :** Développer le sens du nombre.

**Résultat d’apprentissage spécifique**

RAS 2. Déterminer la racine carrée approximative d’un nombre qui n’est pas un carré parfait (se limitant aux nombres entiers positifs).

[C, CE, L, R, T]

[TIC : P2-3.4]

Mathématiques M-9 – Programme d’études de l’Alberta

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**Des fractions au milieu**

Solutions

We had lots of interesting responses to this problem. Ellie and Georgia from Melbourn Village College had a systematic way of finding fractions:

58=76158  to four decimal places.
56=74833  to four decimal places.
We then converted the decimals between56  and 58  into fractions.
For example, 755= $\frac{755}{100}$ . We simplified it by dividing the denominator and the numerator by 5, because 5 goes into both numbers. So one correct anwser is $\frac{151}{20}$.

Ellie and Georgia went on to find other fractions by converting decimals between 7.4833 and 7.6158.

Ashley from Durrington High School and Ha Young Jung from Wesley College used a similar method to Ellie and Georgia to find lots of fractions between 56 and 58 .

John from Takapuna Grammar School showed that all denominators apart from 1 and 3 will work (although he didn't explain how he knew 1 and 3 wouldn't work):

First we calculate 56  and 58 , which turn out to be about 7483 and 7615 respectively. We can also eliminate quite a lot of denominators considering that any number x in which $\frac{1}{x}$58−56  (which is about 0132) has to have a fraction between the two numbers.

Therefore we only need to test 123456 and 7.

24 and 6 can be easily explained off since $\frac{15}{2}$ (or 75) is between 7483 and 7615, and $\frac{15}{2}$ is the same as $\frac{30}{4}$ and $\frac{45}{6}$. 5 is also explained off because $\frac{38}{5}$ (or 76) is also between 7483 and 7615. $\frac{53}{7}$ lies in the range.

Eduardo from the British School in Manila explained clearly why a denominator of 1 wouldn't work:
56 $\frac{p}{q}$58  so 56q2p258q2
We know the denominator 1 will not work because there is no square number between 56 and 58, so when q=1, there is no value of p to satisfy the inequality.

Harry from the Beacon School used a similar algebraic approach to give a very clear and full solution.
Let the fraction be $\frac{x}{y}$.

Now 56$\frac{x}{y}$58
56 $\frac{x^{2}}{y^{2}}$58
56y2x258y2

Therefore, as x and y are whole numbers, there must be a square between 56y2 and 58y2 for a fraction between 56  and 58  to exist for denominator y.

If there is no square in this interval, then the difference between the largest square below 56y2 and the smallest square above 58y2 must be greater than 2y2.

The square root of the former must be the largest integer below 56y2

The difference between n2 and the square immediately above it is:
(n+1) 2−n2=2n+1

So there is a solution in the interval when 2n+12y2, where n is the largest integer below 56y2 .

256y2+12y2  for y=8 but not y=7, so for y8 there will always be a square between 56y2 and 58y2.

For y=1, 56y2=56 and 58y2=58. There are no squares in this range, so 1 cannot be the denominator.

For y=2, 56y2=224 and 58y2=232. There is a square in this range (225=152), so 2 can be the denominator. The numerator x is 225  so the fraction is $\frac{15}{2}$

For y=3, 56y2=504 and 58y2=522. There are no squares in this range, so 3 cannot be the denominator.

For y=4, 56y2=896 and 58 y2=928. There is a square in this range (900=302), so 4 can be the denominator. The fraction is $\frac{30}{4}$, which is equal to $\frac{15}{2}$.

For y=5, 56y2=1400 and 58y2=1450. There is a square in this range (1444=382), so 5 can be the denominator. The fraction is $\frac{38}{5}$.

For y=6, 56y2=2016 and 58y2=2088. There is a square in this range (2025=452), so 6 can be the denominator. The fraction is $\frac{45}{6}$, which is equal to $\frac{15}{2}$.

For y=7, 56y2=2744 and 58y2=2842. There is a square in this range (2809=532), so 7 can be the denominator. The fraction is $\frac{53}{7}$.

There is a fraction between 56  and 58  for all whole denominators except 1 and 3.

Well done Harry!